- Spherical and Solid Harmonics;
- Laguerre Polynomials;
- Chebyshev Polynomials;
- Gegenbauer Polynomials;
- Euler Polynomials;
- Bernoulli Polynomials.
- Jacobi Elliptic Functions and Integrals;
- 3j symbols, 6j symbols and Clebsch Gordan coefficients;

Author: Chris Cannam, with contributions from Winfried Neun, Herbert Melenk, Victor Adamchik, Francis Wright and several others.

16.65 SPECFN2: Package for special special functions

This package provides algebraic manipulations of generalized hypergeometric functions and Meijer's G function. Generalized hypergeometric functions are simplified towards special functions and Meijer's G function is simplified towards special functions or generalized hypergeometric functions.
Author: Victor Adamchik, with major updates by Winfried Neun.
The (generalised) hypergeometric functions

$$ {}_pF_q\left(\left.\begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_q \end{matrix}\right| z\right) $$

are defined in textbooks on special functions as

$$ {}_pF_q\left(\left.\begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_q \end{matrix}\right| z\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n}{(b_1)_n \ldots (b_q)_n} \frac{z^n}{n!} $$

w where $(a)_n$ is the Pochhammer symbol

$$ (a)_n = \prod_{k=0}^{n-1} (a+k) $$

The function

$$ G_{pq}^{mn}\left(z \left| \begin{matrix} (a_p) \\ (b_q) \end{matrix}\right.\right) $$

has been studied by C. S. Meijer beginning in 1936 and has been called Meijer's G function later on. The complete definition of Meijer's G function can be found in [1]. Many well-known functions can be written as G functions, e.g. exponentials, logarithms, trigonometric functions, Bessel functions and hypergeometric functions.
Several hundreds of particular values can be found in [1].

16.65.1 REDUCE operator HYPERGEOMETRIC

The operator `hypergeometric` expects 3 arguments, namely the list of upper parameters (which may be empty), the list of lower parameters (which may be empty too), and the argument, e.g the input:

```
hypergeometric ({},{},z);
```

yields the output

```
 z
e
```

and the input

```
hypergeometric ({1/2,1},{3/2},-x^2);
```

gives

```
 atan(abs(x))
--------------
    abs(x)
```

16.65.2 Extending the HYPERGEOMETRIC operator

Since hundreds of particular cases for the generalised hypergeometric functions can be found in the literature, one cannot expect that all cases are known to the `hypergeometric` operator. Nevertheless the set of special cases can be augmented by adding rules to the REDUCE system, *e.g.*

```
let {hypergeometric({1/2,1/2},{3/2},-(~x)^2) => asinh(x)/x};
```

16.65.3 REDUCE operator `meijerg`

The operator `meijerg` expects 3 arguments, namely the list of upper parameters (which may be empty), the list of lower parameters (which may be empty too), and the argument.
The first element of the lists has to be the list of the first n or m respective

parameters, e.g. to describe

$$G_{11}^{10}\left(x \,\middle|\, \begin{matrix} 1 \\ 0 \end{matrix}\right)$$

one has to write

```
MeijerG({{},1},{{0}},x); % and the result is:

 sign( - x + 1) + sign(x + 1)
------------------------------
              2
```

and for

$$G_{02}^{10}\left(\frac{x^2}{4} \,\middle|\, \begin{matrix} \\ 1+\frac{1}{4}, 1-\frac{1}{4} \end{matrix}\right)$$

```
MeijerG({{}},{{1+1/4},1-1/4},(x^2)/4) * sqrt pi;


                    2                     2
 sqrt(pi)*sqrt(-----------)*sin(abs(x))*x
               abs(x)*pi
-------------------------------------------
                     4
```

Bibliography

[1] A. P. Prudnikov, Yu. A. Brychkov, O. I. Marichev, *Integrals and Series, Volume 3: More special functions*, Gordon and Breach Science Publishers (1990).

[2] R. L. Graham, D. E. Knuth, O. Patashnik, *Concrete Mathematics*, Addison-Wesley Publishing Company (1989).