16.34 LAPLACE: Laplace transforms

This package can calculate ordinary and inverse Laplace transforms of expressions. Documentation is in plain text.

Authors: C. Kazasov, M. Spiridonova, V. Tomov.

Reference: **Christomir Kazasov**, Laplace Transformations in REDUCE 3, Proc. Eurocal '87, Lecture Notes in Comp. Sci., Springer-Verlag (1987) 132-133.

Some hints on how to use to use this package:

Syntax:

`LAPLACE` $(< exp >, < var - s >, < var - t > \ )$

`INVLAP` $(< exp >, < var - s >, < var - t >)$

where $< exp >$ is the expression to be transformed, $< var - s >$ is the source variable (in most cases $< exp >$ depends explicitly of this variable) and $< var - t >$ is the target variable. If $< var - t >$ is omitted, the package uses an internal variable lp!& or il!&, respectively.

The following switches can be used to control the transformations:

`lmon`: If on, sin, cos, sinh and cosh are converted by `LAPLACE` into exponentials,

`lhyp`: If on, expressions $e^{\tilde{x}}$ are converted by `INVLAP` into hyperbolic functions sinh and cosh,

`ltrig`: If on, expressions $e^{\tilde{x}}$ are converted by `INVLAP` into trigonometric functions sin and cos.

The system can be extended by adding Laplace transformation rules for single functions by rules or rule sets. In such a rule the source variable MUST be free, the target variable MUST be il!& for `LAPLACE` and lp!& for `INVLAP` and the third parameter should be omitted. Also rules for transforming derivatives are entered in such a form.

Examples:

```
let {laplace(log(~x),x) => -log(gam * il!&)/il!&,

invlap(log(gam * ~x)/x,x) => -log(lp!&)};


operator f;

let{

laplace(df(f(~x),x),x) => il!&*laplace(f(x),x) - sub(x=0,f(x)),

laplace(df(f(~x),x,~n),x) => il!&**n*laplace(f(x),x) -

for i:=n-1 step -1 until 0 sum

sub(x=0, df(f(x),x,n-1-i)) * il!&**i

when fixp n,

laplace(f(~x),x) = f(il!&)

};
```

Remarks about some functions:

The DELTA and GAMMA functions are known.
ONE is the name of the unit step function.
INTL is a parametrized integral function

$$\texttt{intl}(< expr >, < var >, 0, < obj.var >)$$

which means "Integral of $< expr >$ wrt. $< var >$ taken from 0 to $< obj.var >$",
e.g. `intl(2*y^2, y, 0, x)` which is formally a function in x.

We recommend reading the file LAPLACE.TST for a further introduction.